Biologically Inspired Algorithms

Ant Colony Optimization

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History and Use

- Introduced in 1992
- Author: Marco Doringo
- Based on the behavior of the real ant colonies
- Originally applied to Travelling Salesman Problem (TSP)
- Developed to solve discrete optimization problems

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Inspiration by Real Ants

- Each ant produces **pheromones** while travelling from the nest to food and from the place with the food to the nest → type of communication with the other ants from the colony
- The movement of the ants is random, however, during time, their decision is influenced by the pheromones
- On the shortest path, the pheromones are accumulated
- Important feature of pheromones: **vaporization**

[1]

Principle

- Thanks to vaporization, the paths which are not the shortest are not so attractive for the ants

- In the TSP:
    - The probability that an ant k at node r will choose the destination node s is

$$p_k(r,s) = \begin{cases} \dfrac{\tau(r,s)^{\alpha}\eta(r,s)^{\beta}}{\sum_{u \in M_k} \tau(r,u)^{\alpha}\eta(r,u)^{\beta}}, for\ s \epsilon M_k \\ 0, otherwise \end{cases}$$

[1]

Principle

pheromone → $\tau(r,s)^\alpha$; $\eta(r,s)^\beta$ ← $\frac{1}{d(r,s)}$

$$p_k(r,s) = \begin{cases} \frac{\tau(r,s)^\alpha \eta(r,s)^\beta}{\sum_{u \in M_k} \tau(r,u)^\alpha \eta(r,u)^\beta}, for\ s \epsilon M_k \\ 0, otherwise \end{cases}$$

- where:
  - α … the degree of importance of pheromone
  - β … degree of importance of the distance
  - $u \in M_k$ … is a choice that belongs ant k (neighborhood) when he was at no r
  - Neighborhood of ant k at node r contains all the nodes thatare incident with the node r except already visited nodes

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Principle of Pheromones recalculation I

- When all ants finished their journey, the pheromones must be recalculated:
  1. Vaporization

$$\tau_{r,s} = \tau_{r,s} * \rho$$

← vaporization coefficient

  2. Calculation with pheromones left by all ants during their journey

$$\tau_{r,s} = \tau_{r,s} + \frac{Q}{f(s)}$$

Where :

Q … is a constant (usually 1)

f … the value of the objective function of the solution s

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Principle of Pheromones recalculation II

- When ant k passing the edge, it will leave a pheromone on this edge. The amount of pheromone contained in the segment ij after passed by ant k is given by:

$$\tau_{i,j} \leftarrow \tau_{i,j} + \Delta\tau^{k}$$

where:

$$\Delta\tau_{i,j}^{k} = \begin{cases} \frac{c * f_{best}}{f_{worst}} \; if \; (ij) \in best\ global\ path \\ 0, \; otherwise \end{cases}$$

where :

f_{best} … is the best value of the objective function
c … constant
f_{worst} … is the worst value of the objective function [1]

Principle of Pheromones recalculation II

- With increasing value of pheromone on segment i, j the probability of this segment to be chosen by ants at the next iteration increases

- When a node is passed, then the pheromone evaporation will occur using the following equation:

$$\tau_{i,j} \leftarrow (1-\rho)\tau_{i,j}, \forall (i,j) \in A$$

Where:

$\rho \in [0,1]$ … evaporation rate parameter

A … segments that have been passed by ant k as part of the path from the nest to the food

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Example

Step 1

- 5 cities

- Distance matrix for these cities is:

0	10	12	11	14
10	0	13	15	8
12	13	0	9	14
11	15	9	0	16
14	8	14	16	0

- Matrix of inverse distances (visibility matrix):

0	0.1000	0.0833	0.0909	0.0174
0.1000	0	0.0769	0.0667	0.1250
0.0833	0.0769	0	0.1111	0.0714
0.0909	0.0667	0.1111	0	0.0625
0.0714	0.125	0.0714	0.0625	0

[1]

Example
Step 1

- 5 cities, city 1 is a departure city. Since city 1 is chosen as the beginning , it cannot be chosen again → visibility of the city is **0**

- Initial pheromone matrix

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

- Visibility matrix

0	0.1000	0.0833	0.0909	0.0174
0	0	0.0769	0.0667	0.1250
0	0.0769	0	0.1111	0.0714
0	0.0667	0.1111	0	0.0625
0	0.125	0.0714	0.0625	0

[1]

Example
Step 2

- We calculate the posibility to visit other city using the formula

$$p_k(r,s) = \begin{cases} \frac{\tau(r,s)^\alpha \eta(r,s)^\beta}{\sum_{u\in M_k} \tau(r,u)^\alpha \eta(r,u)^\beta}, for\ s\epsilon M_k \\ 0, otherwise \end{cases}$$

$\tau(1,s)^1\eta(1,s)^2$:
- $1 * 0.1000^2 = 0.0100$
- $1 * 0.0833^2 = 0.0069$
- $1 * 0.0909^2 = 0.0083$
- $1 * 0.07142^2 = 0.0051$

$$\sum_{s\in M_k} \tau(1,s)^1\eta(1,s)^2 = 0.0303$$

- Visibility matrix

0	0.1000	0.0833	0.0909	0.0174
0	0	0.0769	0.0667	0.1250
0	0.0769	0	0.1111	0.0714
0	0.0667	0.1111	0	0.0625
0	0.125	0.0714	0.0625	0

[1]

Example

Step 2

$\tau(1,s)^1\eta(1,s)^2$:

- $1 * 0.1000^2 = 0.0100$
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$$\sum_{s \in M_k} \tau(1,s)^1\eta(1,s)^2 = 0.0303$$

- Visibility matrix

0	0.1000	0.0833	0.0909	0.0174
0	0	0.0769	0.0667	0.1250
0	0.0769	0	0.1111	0.0714
0	0.0667	0.1111	0	0.0625
0	0.125	0.0714	0.0625	0

The probabilities:

City 1 to 2: $\frac{0.01}{0.0303} = 0.3299$

City 1 to 3: $\frac{0.0069}{0.0303} = 0.2291$

City 1 to 4: $\frac{0.0083}{0.0303} = 0.2727$

City 1 to 5: $\frac{0.0051}{0.0303} = 0.1683$

[1]

Example

Step 2

- The probabilities:

City 1 to 2: $\frac{0.01}{0.0303} = 0.3299$

City 1 to 3: $\frac{0.0069}{0.0303} = 0.2291$

City 1 to 4: $\frac{0.0083}{0.0303} = 0.2727$

City 1 to 5: $\frac{0.0051}{0.0303} = 0.1683$

- The cumulative numbers of these probabilities:
  - City 2: 0.3299
  - City 3: 0.5590
  - City 4: 0.8317
  - City 5: 1

[1]

Example

Step 3

- The cumulative numbers of these probabilities:

  - City 2: 0.3299
  - City 3: 0.5590
  - City 4: 0.8317
  - City 5: 1

- Generate random number $r \in [0,1]$: suppose we have generated number $r = 0.6841$
- Compare r with cumulative numbers:
  - $\mathbf{0.5590 < r < 0.8317} \Rightarrow$ an ant will visit the **city 4**

[1]

Example
Step 4

- The city 4 was visited ⇒ the visibility matrix must be adjusted:

0	0.1000	0.0833	**0**	0.0174
0	0	0.0769	**0**	0.1250
0	0.0769	0	**0**	0.0714
0	0.0667	0.1111	**0**	0.0625
0	0.125	0.0714	**0**	0

- Now, the proces of the calculation of the probability of visiting the neighbor city will be repeated

[1]

Example

Step 5

$\tau(4,s)^1\eta(4,s)^2$:

- $1 * 0.0667^2 = 0.0044$
- $1 * 0.1111^2 = 0.0123$
- $1 * 0.07142^2 = 0.0039$

$$\sum_{s \in M_k} \tau(1,s)^1\eta(1,s)^2 = 0.0207$$

The probabilities:

City 4 to 2: $\frac{0.0044}{0.0207} = 0.2147$

City 4 to 3: $\frac{0.0123}{0.0207} = 0.2291$

City 4 to 5: $\frac{0.0039}{0.0207} = 0.1887$

- Visibility matrix

0	0.1000	0.0833	0	0.0174
0	0	0.0769	0	0.1250
0	0.0769	0	0	0.0714
0	0.0667	0.1111	0	0.0625
0	0.125	0.0714	0	0

[1]

Example

Step 5

- The cumulative numbers of these probabilities:
  - City 2: 0.2147
  - City 3: 0.8113
  - City 5: 1

- Generate random number $r \in [0,1]$: suppose we have generated number $r = 0.4024$. Compare r with cumulative numbers:
  - $\mathbf{0.2147 < r < 0.8113}$ ⇒ an ant will visit the **city 3**

- For now, we have path $1 \ \rightarrow 4 \ \rightarrow 3$

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Example

Step 5

- This processs is repeated until all ants have their own paths

- Remember: Each ant starts from another city

- Suppose that we have the paths as follows:

  - Ant 1: $1 \rightarrow 4 \rightarrow 3 \rightarrow 5 \rightarrow 2 \rightarrow 1$ Total distance: 52
  - Ant 2: $1 \rightarrow 4 \rightarrow 2 \rightarrow 5 \rightarrow 3 \rightarrow 1$ Total distance: 60
  - Ant 3: $1 \rightarrow 4 \rightarrow 5 \rightarrow 2 \rightarrow 3 \rightarrow 1$ Total distance: 60

- Use total distances (objective function evaluation) to recalculate the pheromones on the edges:

$$\tau_{r,s} \leftarrow (1-\rho)\tau_{r,s} + \sum_{k=1}^{N} \Delta\tau_{r,s}^{k}$$

[1]

Example

Step 6

- Use total distances (objective function evaluation) to recalculate the pheromones on the edges:

$$\tau_{r,s} \leftarrow (1-\rho)\tau_{r,s} + \sum_{k=1}^{N} \Delta\tau_{r,s}^{k}$$

ρ… evaporation coefficient equals to 0.5

- Pheromone matrix recalculated based on the Ant 1:

0.5000	0.5000	0.5000	0.5192	0.5000
0.5192	0.5000	0.5000	0.5000	0.5000
0.5000	0.5000	0.5000	0.5000	0.5192
0.5000	0.5000	0.5192	0.5000	0.5000
0.5000	0.5192	0.5000	0.5000	0.5000

$$\Delta\tau_{r,s}^{k} = \frac{Q}{f(s)} = \frac{1}{52}$$

[1]

Example

Step 6

- Use total distances (objective function evaluation) to recalculate the pheromones on the edges:

$$\tau_{r,s} \leftarrow (1-\rho)\tau_{r,s} + \sum_{k=1}^{N} \Delta\tau_{r,s}^{k}$$

ρ… evaporation coefficient equals to 0.5

- Pheromone matrix recalculated based on the Ant 2:

0.5000	0.5000	0.5000	0.5359	0.5000
0.5192	0.5000	0.5000	0.5000	0.5167
0.5167	0.5000	0.5000	0.5000	0.5192
0.5000	0.5167	0.5192	0.5000	0.5000
0.5000	0.5192	0.5167	0.5000	0.5000

$$\Delta\tau_{r,s}^{k} = \frac{Q}{f(s)} = \frac{1}{60}$$

[1]

Example

Step 6

- Use total distances (objective function evaluation) to recalculate the pheromones on the edges:

$$\tau_{r,s} \leftarrow (1-\rho)\tau_{r,s} + \sum_{k=1}^{N} \Delta\tau_{r,s}^{k}$$

ρ… evaporation coefficient equals to 0.5

- Pheromone matrix recalculated based on the Ant 3:

0.5000	0.5000	0.5000	0.5526	0.5000
0.5192	0.5000	0.5167	0.5000	0.5167
0.5334	0.5000	0.5000	0.5000	0.5192
0.5000	0.5167	0.5192	0.5000	0.5167
0.5000	0.5359	0.5167	0.5000	0.5000

$$\Delta\tau_{r,s}^{k} = \frac{Q}{f(s)} = \frac{1}{60}$$

[1]

Example
Step 7

- Repeat the proces until the number of iterations equals to the maximum number of iterations

[1]

Literature

[1] Budi Santosa: Tutorial on Ant Colony Optimization, Institut Teknologi Sepuluh Nopember, ITS, bsantosa.com